Large Deviations

1.1

\[ S_n = \sum_{i=1}^{n} X_i, \quad \mu_n = \frac{1}{n} S_n. \]

As the parameters tend to zero

\[ \mu_n \to \mu \quad \text{w.p.1 as } n \to \infty \]

The central limit theorem holds.

\[ \frac{1}{\sigma \sqrt{n}}(S_n - n\mu) \to Z \quad \text{in distribution as } n \to \infty \]

Consider a set of independent and identically distributed random variables \( Z \).

\[ \{ \mu_n \geq \mu + a \} = \{ S_n \geq n(\mu + a) \} \]

For any constant \( a > 0 \), the probability of exceeding \( a \) increases with \( n \).

\[ \forall, \quad \forall \Rightarrow P \{ \mu_n \geq \mu \geq a \} = P \{ X_1 + \ldots + X_n \geq n a \} \]

\[ \geq P \{ X_{j+1} + \ldots + X_{(j+1)k} \geq ak \quad \text{for all } j = 0, 1, \ldots, (n/k) - 1 \} \]

\[ = [P \{ X_1 + \ldots + X_k \geq ak \}]^{n/k} \]

Consider a set of independent and identically distributed random variables \( X \).

\[ \{ X_1 + \ldots + X_n \geq na \} = \{ e^{\theta X_1 + \ldots + X_n} \geq e^{na} \} \]

\[ \leq e^{-na} \mathbb{E} \left[ e^{\theta(X_1 + \ldots + X_n)} \right] \]

\[ = e^{-na} \left[ \mathbb{E} e^{\theta X_1} \right]^n \]

\[ = [e^{-\theta a} \mathbb{E} e^{\theta X_1}]^n \]

For any constant \( a > 0 \), the probability of exceeding \( a \) increases with \( n \).

\[ e^{-\theta a} \mathbb{E} e^{\theta X_1} \approx (1 - \theta a)(1 + \theta^2 \sigma^2) \]

\[ \theta = (a/\sigma^2)^{-1/2} \]

Consider a set of independent and identically distributed random variables \( X \).

\[ e^{-\theta a} \mathbb{E} e^{\theta X_1} \approx 1 - (a/\sigma)^d \]

With \( d = 1.8 \) and \( (1.9) \), the probability of exceeding \( a \) decreases.
\[ P \left\{ X_1 + \ldots + X_n \geq na \right\} \leq \left[ \inf_{\theta > 0} e^{-\theta a} \mathbb{E} e^{\theta X_1} \right]^n \]

\[ \lim_{n \to \infty} \frac{1}{n} \log \mathbb{P} \left\{ a - \varepsilon < \mu_n < a + \varepsilon \right\} \approx -\ell(a) \]

\[ P \left\{ a - \varepsilon < \mu_n < a + \varepsilon \right\} = e^{-n\ell(a) + o(n)} \]

\[ M(\theta) = \mathbb{E} e^{\theta X_1} \]

\[ \ell(a) = -\log \left\{ \inf_{\theta} e^{-\theta a} M(\theta) \right\} = \sup_{\theta} \{\theta a - \log M(\theta)\} \]

\[ \mathbb{P} \{ \mu_n \geq a \} \leq e^{-n\ell(a)} \]

\[ \mathbb{P} \{ \mu_n \geq a \} \geq e^{-n(\ell(a) + \varepsilon)} \]

\[ \ell(a) = \sup_{\theta} \{\theta a - \log M(\theta)\} = \theta^* a - \log M(\theta^*) \]

\[ \ell(a) = -\log \mathbb{E} e^{\theta(X_1 - a)} \]

\[ G(x) = \frac{1}{M(\theta^*)} \int_{-\infty}^{x} e^{\theta y} dF(y). \]
\[ P \{ X_1 \geq \alpha \} = \int_{-\infty}^{\infty} 1_{y \geq \alpha} dF(y) \]
\[ = \int_{-\infty}^{\infty} 1_{y \geq \alpha} e^{-\theta y} e^{\alpha y} dF(y) \]
\[ = M(\theta^*) \int_{-\infty}^{\infty} 1_{y \geq \alpha} e^{-\theta y} dG(y) . \]

\[ \mathbb{P} \{ \mu_n \geq a \} = \mathbb{P} \{ X_1 + \ldots + X_n \geq na \} \]
\[ = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} 1_{y_1 + \ldots + y_n \geq na} dF(y_1) \ldots dF(y_n) \]
\[ = [M(\theta^*)]^n \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} 1_{y_1 + \ldots + y_n \geq na} e^{-\theta y_1} \ldots e^{-\theta y_n} dG(y_1) \ldots dG(y_n) . \]

\[ \mathbb{P} \{ \mu_n \geq a \} \geq [M(\theta^*)]^n \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} 1_{n(a+\varepsilon') \geq y_1 + \ldots + y_n \geq na} e^{-\theta y_1} \ldots e^{-\theta y_n} dG(y_1) \ldots dG(y_n) \]
\[ \geq [M(\theta^*)]^n e^{-n\theta(a+\varepsilon')} \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} 1_{n(a+\varepsilon') \geq y_1 + \ldots + y_n \geq na} dG(y_1) \ldots dG(y_n) \]
\[ = [M(\theta^*)]^n e^{-n\theta(a+\varepsilon')} \mathbb{P} \{ n(a+\varepsilon') \geq X_1 + \ldots + X_n \geq na \} . \]

\[ \mathbb{E}(X_1 - a)e^{\theta(X_1 - a)} = 0 \]
\[ \mathbb{E}X_1 e^{\theta X_1} = a \mathbb{E}e^{\theta X_1} = aM(\theta^*) \]
\[ \mathbb{E}_G X_1 = a . \]
טרם ב פי 3 אוס ב תחילה על ה-
1
פדר: Aבתוך גזר: זה נמצאת מתקנים יוכẩn ב- \( M(\theta) = e^{\theta^2/2} \) \( e^{\lambda e^{-\lambda}} \lambda a^2 \).

למשנים מ DateFormat109.7.3

למשנים \( \lambda \) \(\ell(a) = a \left( \frac{a}{\lambda} - 1 \right) + \lambda \).

למשנים \( \sum_{i=1}^{n} \{0,1\} \) \(\ell(a) = a \log a + (1 - a) \log(1 - a) + \log 2 \)

א义务教育 בניהו ממקורות חסרים והטטרה על \( M(\theta) \) \( \frac{S_n - \mu}{\sigma \sqrt{n}} \approx b \).

Thrown

THROWED

THROWED

THROWED

THROWED
\[ P\{S_n \geq na\} = \frac{1}{\sqrt{2\pi n[v(n)]^2}} e^{-n\ell(n)}[1 + o(1)]. \]

Steven R. S. Varadhan

Lower semi continuous good rate function

\[ \liminf_{x \to y} I(x) \geq I(y). \]

Any lower semi continuous good rate function \( I \) is the rate function of some GMP.

\[ \{ x : I(x) \leq \alpha \} \]

\[ \lim_{n \to \infty} -\frac{1}{n} \log P_n \{ C \} \leq -\inf \{ I(x) : x \in C \} \]

\[ \lim_{n \to \infty} -\frac{1}{n} \log P_n \{ O \} \geq -\inf \{ I(x) : x \in O \} \]

2.7

Limit theorems of empirical measures for iid \( X_n \).

2.5

\[ M(\theta) < \infty \Rightarrow \lim_{\theta \to 0} \text{Var} X_n \]

\( \beta \text{ (Bahadur-Rao)} \)
The contraction principle:

\[ J(y) \doteq \inf \{ I(x) : f(x) = y \} . \]

The Hilbert space-valued pure jump Markov processes

Concerning the contraction principle:

\[ LDP \text{ to } \{ L_n \} \]

\[ I_\rho(\nu) \doteq \sum_i \nu_i \log \frac{\nu_i}{\rho_i} . \]

\[ \lim_{n \to \infty} \frac{1}{n} \log P \{ L_n \in B^c_n(\rho) \} = -\inf \{ I_\rho(\nu) : \nu \in B_n^c(\rho) \} . \]
 drv nominated we specify a second order stationary M/M/1 queueing system (which we use for the rest of the paper).

\[ \lambda = \lambda e_{1} e_{1} = 1 \] for all \( \lambda_{1} \).

3.4.2 Theoretical results:

\[ \lambda_{2}(x) = \mu_{1} \mathbf{1}_{\{x \neq 0\}} \]

\[ e_{1} = \frac{1}{\lambda_{1}} \]

\[ e_{2} = -1 \]

\[ \lambda_{2} = \lambda \mathbf{1}_{\{x \neq 0\}} \] for all \( \lambda_{1} \).

3.5 Theoretical Results:

\[ x(t) = x(n(t))/n \]

\[ n \lambda_{j}(x) \]

\[ \{\lambda_{2}(x) \} \]

\[ n \lambda_{j}(x) \]

\[ x(n(t))/n \]

\[ \frac{1}{n} \log \frac{a_{n}}{b_{n}} = \frac{1}{n} \log(\frac{a_{n} + b_{n}}{a_{n} + b_{n}}) = a. \]

\[ \alpha \geq \beta \] 4.2

\[ \alpha \geq \beta \] 4.2

\[ \lim_{n \to \infty} \frac{1}{n} \log a_{n} = a \]

\[ \lim_{n \to \infty} \frac{1}{n} \log b_{n} = b. \] 4.1

4.1

\[ \lim_{n \to \infty} \frac{1}{n} \log(\frac{a_{n} + b_{n}}{a_{n} + b_{n}}) = a. \] 4.2

\[ \lim_{n \to \infty} \frac{1}{n} \log(\frac{a_{n} + b_{n}}{a_{n} + b_{n}}) = a. \] 4.2
\[
\frac{1}{n} \log a_n = \frac{1}{n} \log (a_n + b_n) \leq \frac{1}{n} \log a_n (1 + b_n/a_n) \leq \frac{1}{n} \log a_n + \frac{1}{n} \frac{b_n}{a_n} \to \frac{1}{n} \log a_n.
\]

Theorem 4.3: \( n \)th moment exponentially tight \( \{z^n\} \) and 4.4: \( \{x^n\}\) exponentially tight.

\[
\lim_{n \to \infty} \frac{1}{n} \log \mathbb{P}\{z^n \notin K_M\} \leq -M.
\]

Lemma 4.5: \( n \)th moment compactness and 4.4.4: \( \lambda_j(x) \) as \( n \)th moments.\[\sum_j \lambda_j(x) e_j\]

Kurtz 4.5: If \( \lim_{\varepsilon \to 0} C_2(\varepsilon) \) exists for any compact set \( \{x^n(0)\} \) and 4.5.5: \( x^n(0) \) has exponentially fast convergence to 0 for all \( x \) in the finite-dimensional state space.\[\sum \lambda_j(x) e_j(x)\]

\[
\frac{dx^n(t)}{dt} = \sum_j \lambda_j(x^n(t)) e_j, \quad x^n(0) = \lim_n x^n(0).
\]

Kurtz 4.5: If \( \lim_{\varepsilon \to 0} C_2(\varepsilon) \) exists for any compact set \( \{x^n(0)\} \) and 4.5.5: \( x^n(0) \) has exponentially fast convergence to 0 for all \( x \) in the finite-dimensional state space.\[\sum \lambda_j(x) e_j(x)\]

\[
\frac{dx^n(t)}{dt} = \sum_j \lambda_j(x^n(t)) e_j, \quad x^n(0) = \lim_n x^n(0).
\]

Kurtz 4.5: If \( \lim_{\varepsilon \to 0} C_2(\varepsilon) \) exists for any compact set \( \{x^n(0)\} \) and 4.5.5: \( x^n(0) \) has exponentially fast convergence to 0 for all \( x \) in the finite-dimensional state space.\[\sum \lambda_j(x) e_j(x)\]
4.2 calibrated to untruthful transmitters with finite-dimensional parameter estimates, the authors exploited exponentially tight bounds on the error of the estimation.

To avoid trivial cases, the authors considered the case where the estimation error is small enough to be negligible.

The bound for the estimation error is given by

$$\Delta_{t, \ell} = \frac{1}{T} \sum_{k=1}^{T} \Delta_{t, \ell} \left( r(k), x(k) \right) - \int_{0}^{T} \ell(r(t), r'(t)) \, dt \approx -I(r)$$

The authors then considered the case where the estimation error is small enough to be negligible.

The bound for the estimation error is given by

$$\Delta_{t, \ell} = \frac{1}{T} \sum_{k=1}^{T} \Delta_{t, \ell} \left( r(k), x(k) \right) - \int_{0}^{T} \ell(r(t), r'(t)) \, dt \approx -I(r)$$

The authors then considered the case where the estimation error is small enough to be negligible.

The bound for the estimation error is given by

$$\Delta_{t, \ell} = \frac{1}{T} \sum_{k=1}^{T} \Delta_{t, \ell} \left( r(k), x(k) \right) - \int_{0}^{T} \ell(r(t), r'(t)) \, dt \approx -I(r)$$

The authors then considered the case where the estimation error is small enough to be negligible.
size the triplets. \( \ell \) characterizes the fragmented clusters.

Thus, each cluster is represented by an element of the set \( S \), which is defined as:

\[
S = \{ r : r(0) = a, r(T) = b \}
\]

The variational problem is stated as follows:

\[
\min\{ I(r) : r \in S \}
\]

which is to be solved, for \( a \) and \( b \) fixed, in \( \mathbb{R}^d \).

For each cluster, the variational problem is of the form:

\[
\min\{ I(r) : r \in S \}
\]

where \( I(r) \) is a functional that depends on the cluster configuration.

The solution to this problem is obtained through the reflection map and the contraction principle.

The reflection map is given by:

\[
r^* = \begin{cases} r & \text{if } r \cdot e_1 > 0 \\ -r & \text{if } r \cdot e_1 < 0 \end{cases}
\]

where \( e_1 \) is a unit vector.

The contraction principle ensures that the solution is unique and stable.

The variational problem is solved numerically using optimization algorithms.

Finally, the entire process is repeated for each of the fragmented clusters, yielding a complete clustering of the data.
החלף התלheid

5.1

ほかדות ההליך הנפוג ששי התלheid פאסיו, ע"צ פרמטוריס

\[ e_1 = 1, \quad e_2 = -1, \quad \lambda_1 = \lambda, \quad \lambda_2 = \mu \]

מסמטט קורא נקביל מידי את ההרטונג "הפהפיעה" של התלheid:

\[ \frac{d}{dt} x^\infty(t) = \lambda - \mu, \quad x^\infty(0) = x^\infty(0) + (\lambda - \mu) \cdot t \]

כלומר כתרד התלheid מכונסח למלכזים לגירט. ששמו הפונקציה היא דויבי או איש ושייל אוחרו.

על התלheid זה כנף חתפ ב玠וש

\[ \ell(x, y) = \ell(y) = y \log \left( \frac{y + \sqrt{y^2 + 4\lambda \mu}}{2\lambda} \right) + \lambda + \mu - \sqrt{y^2 + 4\lambda \mu} \]

כימו ש-

\[ \ell(\lambda - \mu) = 0 \]

ךביבס (סמלים) ש-

5.2

洞察 תובש את (בז"ל של) הצרכים שלה ב-1. לכל נקף על-זבז

\[ \inf \{ I(r) : r(0) = 0, \quad r(T) \geq a \} \]

כינו ש \( \ell \) בולק זך במשפט המורדה על במלכזים המוקזנים, או ידימס כנד עדכן אפ forall ש-\( \ell \) ש- \( \ell \) נקשר לעבל הדינמיקות של התלheid.

\[ \inf \{ \ell(b/T) : b \geq a \}. \]

אלו ש \( \ell \) נدفاعים, נקוד ה蘼🌄 ש \( \ell \) \( a/T > \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בינו \( \lambda - \mu \) - בино
\[
\frac{2\lambda}{c + \sqrt{c^2 + 4\lambda\mu}} - \frac{1}{2\lambda} \left( 1 + \frac{1}{2} \left[ \sqrt{c^2 + 4\lambda\mu} \right]^{-1} \cdot 2c \right) - \frac{1}{c^2} (\lambda + \mu - \sqrt{c^2 + 4\lambda\mu}) - \frac{1}{2} \left[ \sqrt{c^2 + 4\lambda\mu} \right]^{-1} \cdot 2c
\]

\[
= \frac{1}{c + \sqrt{c^2 + 4\lambda\mu}} \cdot \left( 1 + \left[ \sqrt{c^2 + 4\lambda\mu} \right]^{-1} \cdot c \right) - \frac{1}{c^2} (\lambda + \mu - \sqrt{c^2 + 4\lambda\mu}) - \left[ \sqrt{c^2 + 4\lambda\mu} \right]^{-1} = 0
\]

\[c = \mu - \lambda > 0\]

\[
= \frac{1}{2\mu} \left( 1 + \frac{\mu - \lambda}{\lambda + \mu} \right) - \frac{1}{\lambda + \mu}
\]

\[
= \frac{1}{2\mu} \frac{2\mu}{\lambda + \mu} - \frac{1}{\lambda + \mu} = 0
\]

\[
\inf \left\{ \frac{1}{2} e^{-\ell(c)} : c > 0 \right\} = a \log \frac{\mu}{\lambda}
\]

\[
\text{Ki}\]

\[
\frac{d}{dt} x^\infty(t) = \begin{cases} 
\lambda - \mu & x^\infty(t) > 0 \\
0 & x^\infty(t) = 0 
\end{cases}
\]

\[
x^\infty(t) = \begin{cases} 
x^\infty(0) + (\lambda - \mu) \cdot t & x^\infty(t) > 0 \\
0 & t \geq x^\infty(0)/(\mu - \lambda) 
\end{cases}
\]

\[
= \max \{ x^\infty(0) + (\lambda - \mu) \cdot t, 0 \}
\]

\[x(t) \text{ reflection map } \]

\[\text{LDP} \]

\[\text{Cdi Ley Cellular Automaton LDP} \]

\[R[x](t) = x(t) - \inf_{0 \leq s \leq t} x(s).
\]
\begin{align}
|R[x](t) - R[y](t)| &= |x(t) - y(t) - \inf_{0 \leq s \leq t} x(s) + \inf_{0 \leq s \leq t} y(s)| \\
&\leq |x(t) - y(t)| + \sup_{0 \leq s \leq t} |x(s) - y(s)| \\
&\leq 2 \sup_{0 \leq s \leq T} |x(t) - y(t)| \\
\sup_{0 \leq t \leq T} |R[x](t) - R[y](t)| &\leq 2 \sup_{0 \leq t \leq T} |x(t) - y(t)|.
\end{align}

The contraction mapping theorem:

\begin{align}
I(r) &= \begin{cases} 
\int_0^T \ell(r(t),r'(t)) \, dt & \text{if } r \text{ absolutely continuous} \\
\infty & \text{otherwise}
\end{cases} \\
\ell(x,y) &= \begin{cases} 
\ell_f(y) & \text{if } x > 0 \text{ or } y > 0 \\
0 & \text{if } x = 0 \text{ and } y = 0 \\
\infty & \text{if } x < 0 \text{ or } x = 0 \text{ and } y < 0.
\end{cases}
\end{align}

The contraction mapping theorem applies to the mapping $R$, making $M/M/1$ and the M/D/1 models stable.

\begin{align}
\inf \{I_f(u) : R[u] = r\} &= \inf \int_0^T \ell_f(u(t)) \mathbb{1}_{\{r'(t) > 0\}} \, dt : R[u] = r \\
&= \inf \int_0^T \ell_f(r(t)) \mathbb{1}_{\{r'(t) > 0\}} \, dt : R[u] = r \\
&= \inf \int_0^T \ell(r(t),r'(t)) \, dt.
\end{align}

Theorem 6.1: The mapping $R$ is a contraction mapping theorem, making $M/M/1$ and the M/D/1 models stable.

\begin{align}
\inf \{I_f(u) : R[u] = r\} &= \inf \int_0^T \ell_f(u(t)) \mathbb{1}_{\{r'(t) > 0\}} \, dt : R[u] = r \\
&= \inf \int_0^T \ell_f(r(t)) \mathbb{1}_{\{r'(t) > 0\}} \, dt : R[u] = r \\
&= \inf \int_0^T \ell(r(t),r'(t)) \, dt.
\end{align}

Theorem 6.1: The mapping $R$ is a contraction mapping theorem, making $M/M/1$ and the M/D/1 models stable.

\begin{align}
\inf \{I_f(u) : R[u] = r\} &= \inf \int_0^T \ell_f(u(t)) \mathbb{1}_{\{r'(t) > 0\}} \, dt : R[u] = r \\
&= \inf \int_0^T \ell_f(r(t)) \mathbb{1}_{\{r'(t) > 0\}} \, dt : R[u] = r \\
&= \inf \int_0^T \ell(r(t),r'(t)) \, dt.
\end{align}

Theorem 6.1: The mapping $R$ is a contraction mapping theorem, making $M/M/1$ and the M/D/1 models stable.

\begin{align}
\inf \{I_f(u) : R[u] = r\} &= \inf \int_0^T \ell_f(u(t)) \mathbb{1}_{\{r'(t) > 0\}} \, dt : R[u] = r \\
&= \inf \int_0^T \ell_f(r(t)) \mathbb{1}_{\{r'(t) > 0\}} \, dt : R[u] = r \\
&= \inf \int_0^T \ell(r(t),r'(t)) \, dt.
\end{align}

Theorem 6.1: The mapping $R$ is a contraction mapping theorem, making $M/M/1$ and the M/D/1 models stable.

\begin{align}
\inf \{I_f(u) : R[u] = r\} &= \inf \int_0^T \ell_f(u(t)) \mathbb{1}_{\{r'(t) > 0\}} \, dt : R[u] = r \\
&= \inf \int_0^T \ell_f(r(t)) \mathbb{1}_{\{r'(t) > 0\}} \, dt : R[u] = r \\
&= \inf \int_0^T \ell(r(t),r'(t)) \, dt.
\end{align}

Theorem 6.1: The mapping $R$ is a contraction mapping theorem, making $M/M/1$ and the M/D/1 models stable.
\( I(\hat{r}) \leq I(r) \) \( \forall r(x, y) = x \), \( r(T) = y \)

\( r(t) \geq 0 \) \( \forall t \leq t_1 < t_p \leq t_2 < T \).

\[ t_p = \sup \{ t < T : r(t) = 0 \} \]

\[ t_1 = \inf \{ t > 0 : r(t) = 0 \} \]

\( -0.5 \leq r(t) \leq 0 \)

\[ \int_0^T (r(t), r'(t)) \, dt \]

\( \int_0^{t_1} (r(t), r'(t)) \, dt \]

\( \int_{t_1}^{t_2} (r(t), r'(t)) \, dt \]

\( \int_{t_2}^T (r(t), r'(t)) \, dt \]

\[ \int_0^T \dot{r}(t) \, dt \]

\[ \int_0^{t_1} \dot{r}(t) \, dt \]

\[ \int_{t_1}^{t_2} \dot{r}(t) \, dt \]

\[ \int_{t_2}^T \dot{r}(t) \, dt \]

\[ \int_0^T r(t) \, dt \]

\[ \int_0^{t_1} r(t) \, dt \]

\[ \int_{t_1}^{t_2} r(t) \, dt \]

\[ \int_{t_2}^T r(t) \, dt \]

\[ \int_0^T (r(t), r'(t)) \, dt \]

\[ \int_0^{t_1} (r(t), r'(t)) \, dt \]

\[ \int_{t_1}^{t_2} (r(t), r'(t)) \, dt \]

\[ \int_{t_2}^T (r(t), r'(t)) \, dt \]

\[ \int_0^T r(t) \, dt \]

\[ \int_0^{t_1} r(t) \, dt \]

\[ \int_{t_1}^{t_2} r(t) \, dt \]

\[ \int_{t_2}^T r(t) \, dt \]

\[ \int_0^T (r(t), r'(t)) \, dt \]

\[ \int_0^{t_1} (r(t), r'(t)) \, dt \]

\[ \int_{t_1}^{t_2} (r(t), r'(t)) \, dt \]

\[ \int_{t_2}^T (r(t), r'(t)) \, dt \]

\[ \int_0^T r(t) \, dt \]

\[ \int_0^{t_1} r(t) \, dt \]

\[ \int_{t_1}^{t_2} r(t) \, dt \]

\[ \int_{t_2}^T r(t) \, dt \]
Theorem 1

\[ P\{S_n \geq na\} \approx e^{-n\theta(a)} \]

where \( \theta(a) \) is a constant.

Corollary 1

\[ P\{S_j \geq ja\ for all \ j \leq n\} \approx e^{-n\theta(a)} \]

The calculus of variations is the simplest problem of the form

\[ \int_0^T \ell(r(t), r'(t)) \, dt \]

where \( \ell \) is a functional that shall satisfy certain conditions. The functional \( \ell \) is defined on the space of functions \( r \) that are twice continuously differentiable on \( [0, T] \) and satisfy certain boundary conditions.
\[ F(\delta) = \int_0^T \ell(r(t) + \delta x(t), r'(t) + \delta x'(t)) \, dt \]

The Euler necessary condition is

\[ 0 = \frac{d}{d\delta} \int_0^T \ell(r(t) + \delta x(t), r'(t) + \delta x'(t)) \, dt \bigg|_{\delta=0} \]

\[ = \int_0^T \left[ x(t) \frac{d}{dr} \ell(r(t), r'(t)) + x'(t) \frac{d}{dr'} \ell(r(t), r'(t)) \right] \, dt \]

\[ = \int_0^T x(t) \left[ \frac{d}{dr} \ell(r(t), r'(t)) - \frac{d}{dt} \frac{d}{dr'} \ell(r(t), r'(t)) \right] \, dt \]

The necessary condition for the minimum is

\[ \frac{d}{dr} \ell(r(t), r'(t)) - \frac{d}{dt} \frac{d}{dr'} \ell(r(t), r'(t)) = 0. \]

List of references:

A. Shwartz, A. Weiss, Large deviations for performance evaluation, CRC 1995, second edition. [1]